

PII: S0301-9322(96)00034-1

 Int. J. Multiphase Flow Vol. 22, No. 5, pp. 1031–1034, 1996

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 00034-1
 0301-9322/96 \$15.00 + 0.00

# **BRIEF COMMUNICATION**

# VELOCITY OF LONG BUBBLES IN OSCILLATING VERTICAL PIPES

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(Received 4 December 1995; in revised form 8 May 1996)

### 1. INTRODUCTION

The velocity of large gas bubbles in stagnant liquids has been investigated by many authors, starting with the theoretical work of Dumitrescu (1943) and Davies & Taylor (1950). The initial theoretical analysis has been extended by subsequent investigators, and a recent comprehensive review indicates the latest analytical developments (Clarke & Issa 1993). The results for the rise velocity,  $U_0$ , of long gas bubbles in stagnant liquids in vertical pipes are

$$U_0 = k_0 (gD)^{0.5}$$
 [1]

where g is the gravitational acceleration, D is the internal pipe diameter and the constant  $k_0 = 0.35$ .

The subsequent theoretical and experimental work has considered the influence of various additional parameters, such as pipe inclination and the liquid properties (Zukoski 1966) and the length of the gas bubbles (Clarke & Issa 1993). However, in many situations when long bubbles or slug flows occur the pipes undergo severe vibrations which may effect the rise velocity of the long bubbles. It is well known that the behaviour of particles or small bubbles is affected by liquid oscillations and that their rise or settling velocities may be significantly reduced (Boyadzhiev 1973; Herringe 1976; Kubie 1980).

It is the purpose of this paper to investigate the rise velocity of large gas bubbles in oscillating vertical pipes, by subjecting the pipes to a sinusoidal vertical motion along their axis. Vertical pipe vibrations have been observed when slug flow from vertical pipes discharges into partially filled horizontal tanks. Standing waves on the interface in the tank can result in vertical vibrations of the tank and the attached pipe.

## 2. EXPERIMENTAL WORK

An apparatus has been designed which enables movement of a vertical pipe in an exact sinusoidal vertical motion. The apparatus is sufficiently flexible to allow for changes in the amplitude and the frequency of the angular motion. A diagram of the experimental apparatus is shown in figure 1. The major components are: a rigid frame, flywheel, scotch yoke mechanism and bearings, vertical pipe holder and vertical pipes. A flywheel, powered by an electric motor via a gearbox, was used to vary the amplitude of the vertical motion and to improve the dynamic stability of the rig. This flywheel, 500 mm in diameter, was drilled with a set of holes to which the pivot holder of the horizontal linear bearing was attached. This allowed for the amplitude of the vertical motion to be varied between 5 mm and 200 mm. In order to obtain a sinusoidal vertical motion a Scotch yoke mechanism was used. A pivot, connected to the flywheel, was inserted into a roller bearing. The roller bearing was then bolted onto a sliding member of a horizontal linear

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Figure 1. Diagram of the experimental apparatus.

bearing, which was connected to the pipe holder. As the flywheel started to spin a vertical sinusoidal motion was obtained. The vertical pipe holder slides on a vertical linear bearing fixed to the frame.

Two perspex pipes, 2000 mm long with internal diameter D of 22 and 44 mm, respectively, and sealed at the top, were used in the experiments. The pipes were filled with the working fluid (tap water at room temperature) and closed with a stopper. The influence of the following parameters of the sinusoidal motion were investigated: amplitude, A of 50, 100 and 200 mm and acceleration  $A\omega^2$  of 0 (stationary vertical pipe), 1, 5, 10 and 15 m s<sup>-2</sup>. The experimental apparatus did not allow investigation of higher accelerations. The required amplitude A, and angular velocity,  $\omega$ , of the sinusoidal motion were set and the electric motor was started. When steady-state conditions were reached the stopper was released and a long air bubble started to rise in the vertical pipe. Different ways of removing the stopper were investigated but they had no noticeable effect on the rise velocity of the bubbles.

The pipes were graduated at 100 mm intervals and the time taken for the bubbles to rise between 300 mm and 1800 mm was taken to determine the average bubble rise velocity. The average bubble velocity was always calculated over at least five experimental runs. It is estimated that the maximum error in determining the bubble velocity was less than about 5%. Finally, video recordings were made to study the shape of the bubbles.

## 3. RESULTS AND DISCUSSION

The bubble rise velocity in stationary pipes is, as expected, in agreement with the results of previous studies. It was observed that for small values of the acceleration (up to  $10 \text{ m s}^{-2}$ ), the acceleration had no noticeable effect on the shape of the bubbles. However, as the acceleration increased to  $15 \text{ m s}^{-2}$  the bubbles started to distort. The bubble nose became more elongated and its curvature increased. As the acceleration increased beyond  $15 \text{ m s}^{-2}$  the bubbles started to break up.

Experimental results indicate that it is the acceleration and not the amplitude which influences the behaviour of bubbles in oscillating pipes, and that the bubble velocity, U, is significantly reduced as the acceleration increases. The experimental results for the velocity ratio  $U/U_0$  are plotted against the relative acceleration  $a = A\omega^2/g$  in figure 2 for the two pipe diameters



Figure 2. A plot of the velocity ratios  $U/U_0$  and  $U_D/U_0$  vs the relative acceleration,  $A\omega^2/g$ : experimental data for D = 22 mm ( $\blacksquare$ ), experimental data for D = 44 mm ( $\blacktriangle$ ), theoretical results for  $U/U_0$  (solid line) and theoretical results for  $U_D/U_0$  (dashed line).

investigated. Figure 2 demonstrates the decrease of  $U/U_0$  with increasing relative acceleration, and also shows that this decrease is somewhat more significant for the smaller diameter pipe. However, it should be noted that this is within the experimental error discussed above.

Since it is the acceleration acting on the system which affects the bubble velocity in oscillating pipes, it is assumed that it is the effective acceleration which should be used instead of the gravitational acceleration in [1], and that this effective acceleration can be used to determine the instantaneous bubble velocity. It will be assumed that the effective acceleration,  $g_E$ , is the sum of the gravitational acceleration and  $A\omega^2 \sin \omega t$ , provided this sum is not less than zero and hence

$$g_{\rm E} = \max[(g + A\omega^2 \sin \omega t), 0].$$
<sup>[2]</sup>

The instantaneous bubble velocity,  $U_{\rm I}$ , is then given as

$$U_1 = k_0 (g_E D)^{0.5}$$
 [3]

and the average bubble velocity as

$$U = \frac{1}{T} \int_0^T U_1 \,\mathrm{d}t \tag{4}$$

where T is the periodic time.

Finally, [1]-[4] can be re-arranged as

$$\frac{U}{U_0} = \frac{1}{T} \int_0^T \max[(1 + a \sin \omega t), 0] \, \mathrm{d}t.$$
 [5]

The theoretical results are compared with the experimental data in figure 2. The figure shows that whereas the trends are identical, the theoretical results underpredict the reduction in  $U/U_0$  observed experimentally. This is probably due to the assumption of quasi-steady behaviour of the bubbles implied in the use of the effective acceleration to derive the instantaneous bubble velocity.

Nevertheless, there could be another reason for this underprediction. As discussed above, as the acceleration increases the bubble nose becomes more elongated and its radius of curvature decreases, and the bubbles will eventually break up. The bubble distortions are very small for low relative accelerations, but become more pronounced as the relative acceleration a increases above about 1.5. The altered shape, and hence the decreased radius of curvature of the bubble nose, can

be modelled by assuming that, as the relative acceleration increases, the bubbles are confined in a pipe with, effectively, a smaller internal diameter,  $D_D$ . To investigate this effect a simple assumption is made, which relates  $D_D$  to the actual pipe diameter D:

$$\frac{D_{\rm D}}{D} = \left(1 - \frac{a}{a_{\rm C}}\right)^n \tag{6}$$

where  $a_c$  is the critical relative acceleration at which the bubble is completely broken up (and  $D_D$  is effectively zero). The rise velocity of such distorted bubbles,  $U_D$ , can be then obtained from the above equations as

$$\frac{U_{\rm D}}{U_0} = \left(1 - \frac{a}{a_{\rm C}}\right)^{n/2} \frac{U}{U_0}$$
[7]

where  $U/U_0$  is given by [5]. Typical results of [7] with the critical relative acceleration,  $a_c$ , of 1.7 and n = 0.05 are also shown in figure 2, which indicates a reasonable agreement between the theoretical approach developed in this paper and the experimental data.

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